4.10-4.15 Test Review KEY

Name _____ **Date** _____ **Period** _____

1. Find $f \circ g(x)$ $\quad f(x)=\sqrt{x+2} \quad g(x)=2x^2+1$

$f(g(x)) = f(2x^2+1) = \sqrt{2x^2+1+2}$

$= \sqrt{2x^2+3}$

2. $f(x)=\frac{x+1}{x-1} \quad g(x)=\frac{1}{x}$ *Find domain of* $f \circ g$.

$x \neq 0$

$(f \circ g)(x) = \frac{\frac{1}{x}+1}{\frac{1}{x}-1} \cdot \frac{x}{x} = \frac{1+x}{1-x}$ $\quad x \neq 1$

$(-\infty, 0) \cup (0, 1] \cup (1, \infty)$

3. What makes a function one-to-one?

A function where every element in the range of f correspond with exactly one element in the domain.

4. Are the following functions inverses? (Need to show f(g(x)) = x and g(f(x)) = x)

$f(x)=3x-6 \qquad g(x)=\frac{1}{3}x+2$

$f(g(x)) = 3(\frac{1}{3}x+2)-6 = x+6-6 = x$ ✓

$g(f(x)) = \frac{1}{3}(3x-6)+2 = x-2+2 = x$ ✓

5. Find the inverse of the function. State the domain and range of f. $f(x)=\frac{2x+3}{5x-2}$

Domain: $x \neq 2/5$ $(-\infty, 2/5) \cup (2/5, \infty)$

Range: $y \neq \frac{2}{5}$ $(-\infty, 2/5) \cup (2/5, \infty)$

$(5y-2)x = \frac{2y+3}{5y-2}(5y-2)$

$5xy-2x = 2y+3$

$5xy-2y = 2x+3$

$y(5x-2) = 2x+3$

$f^{-1}(x) = \frac{2x+3}{5x-2}$

6. $f(x)=\frac{2}{2x-5}$ *State the domain and range of* f *and* f^{-1}

f: Domain: $(-\infty, 5/2) \cup (5/2, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

f^{-1}: Domain: $(-\infty, 0) \cup (0, \infty)$

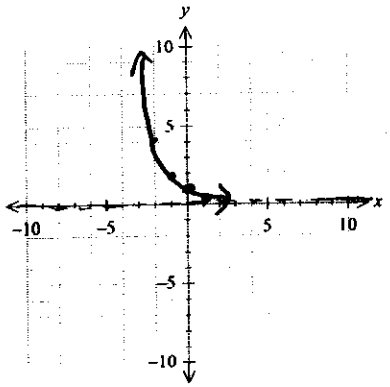
Range: $(-\infty, 5/2) \cup (5/2, \infty)$

7. Find $5^{3.2}$ ≈ 172.466

8. Graph $\left(\frac{1}{2}\right)^x$ Identify the intercepts, asymptotes, domain and range.

x	y
-1	2
0	1
1	1/2

y-int: (0,1)
x-int: none
Asympt: $y=0$
Domain: $(-\infty,\infty)$
Range: $(0,\infty)$



9. Evaluate the following expressions.

a) $\log_8 \frac{1}{64} = -2$ b) $\ln e = 1$ c) $\log_{12} 1 = 0$

10. Find the domain of $f(x) = \ln(10 - x)$. Show work!

$10 - x > 0$
$-x > -10$
$x < 10$
$(-\infty, 10)$

11. Solve the following equations. Show work!

a) $\log_3(x-4) = 2$

$3^2 = x - 4$
$9 = x - 4$
$13 = x$

b) $\log_2(x^2 - 2x) = 3$

$2^3 = x^2 - 2x$
$8 = x^2 - 2x$
$0 = x^2 - 2x - 8$
$0 = (x-4)(x+2)$
$x = 4 \quad x = -2$

c) $e^{3x} = 10$

$\frac{\ln 10}{3} = \frac{3x}{3}$
$x = \frac{\ln 10}{3}$

d) $\log(x+6)=2$

$10^2 = x+6$

$100 = x+6$

$\boxed{94 = x}$

e) $\log_4(x+5)=3$

$4^3 = x+5$

$64 = x+5$

$\boxed{59 = x}$

f) $3^{(2x-5)}=9$

$3^{2x-5} = 3^2$

$2x-5 = 2$

$2x = 7$

$\boxed{x = 7/2}$

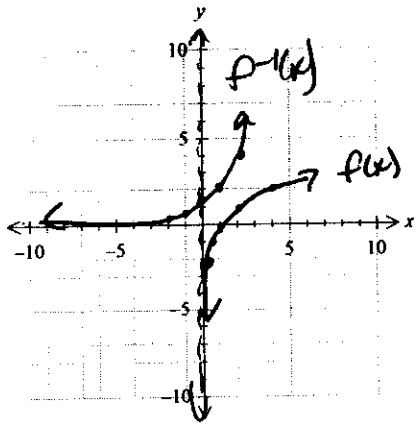
12. Graph the function and its inverse on the same Cartesian plane. Label each function on the graph and make a table of five key points for each function.

$f(x)=\log_2 x$

x	y
1/4	-2
1/2	-1
1	0
2	1
4	2

$f^{-1}(x) = 2^x$

x	y
-2	1/4
-1	1/2
0	1
1	2
2	4



13. Write as the sum and/or difference of logarithms. Express powers as factors. $\log_8\left(\frac{2x-3}{x^4}\right)$

$\log_8(2x-3) - 4\log_8 x$

14. Express as a single logarithm. $2\log_4 3+\frac{1}{2}\log_4(x-5)-\frac{1}{3}\log_4 x$

$\log_4\left(\frac{3^2\sqrt{x-5}}{\sqrt[3]{x}}\right)$ or $\log_4\left(\frac{9\sqrt{x-5}}{\sqrt[3]{x}}\right)$

15. Use the change of base formula and a calculator to evaluate the logarithm. Round your answer to three decimal places. $\log_{3.4} 210$

$\frac{\log 210}{\log 3.4} \approx 4.369$

16. Find the amount which results from the following investment. $10,000 invested at 8% compounded quarterly after a period of 5 years.

$A = 10{,}000\left(1+\frac{.08}{4}\right)^{(4\cdot 5)} = \$14{,}859.47$

17. The formula for a small bacteria population is $P(t) = 400e^{.23t}$ After how many years will the population reach 2000?

$\frac{2000}{400} = \frac{400e^{.23t}}{400}$

$5 = e^{.23t}$

$\frac{\ln 5}{.23} = \frac{.23t}{.23}$

$t \approx 7 \text{ years}$

18. The half-life of Wellsonium is 630 years. If 50 grams are present now how much will be present in 800 years?

$\frac{1}{2} = e^{k(630)}$

$\frac{\ln(1/2)}{630} = \frac{630k}{630}$

$-.0011 \approx k$

$A(800) = 50e^{(-.0011)(800)}$

$\approx 20.74 \text{ grams}$